

INDEX OF A FINITISTIC SPACE AND A GENERALIZATION OF THE TOPOLOGICAL CENTRAL POINT THEOREM

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ABSTRACT. In this paper we prove that if G is a p -torus (resp. torus) group acting without fixed points on a finitistic space X (resp. with finitely many orbit types), then the G -index $i_G(X) < \infty$. Using this G -index we obtain a generalization of the Central Point Theorem and also of the Tverberg Theorem for any d -dimensional Hausdorff space.

1. INTRODUCTION

Let X be a topological space with an involution T and let \mathbb{S}^n denote the n -sphere with antipodal involution. Then the \mathbb{Z}_2 -index of the \mathbb{Z}_2 -space X is defined as the smallest integer n such that there is an equivariant map $X \rightarrow \mathbb{S}^n$ (see [9], p.95). Then the Borsuk-Ulam theorem asserts that the \mathbb{Z}_2 -index of \mathbb{S}^n is n . The idea of this \mathbb{Z}_2 -index has been generalized in many ways, first for any finite group (see [4]) and then for arbitrary G -spaces where G is a compact Lie group acting on a space X (see [9], [13]). Its multiple applications in proving several nonembedding results like the van Kampen-Flores theorem, theorems like Central Point theorem and the Tverberg theorem etc. make it a powerful tool in topological combinatorics. The idea of an "ideal-index" of a G -space X , using the spectral sequence arising from the principal G -bundle map of Borel construction, has proved to be quite effective (see [13], [8]). This ideal-index defines very naturally several numerical indices, in particular the numerical G -index $i_G(X)$ of a G -space X . If $f : X \rightarrow Y$ is a G -map then it follows easily that $i_G(X) \leq i_G(Y)$. This result, combined with the G -index of the deleted join with natural group actions, is utilized in showing the nonexistence of a G -map from a G -space X to a G -space Y . However, in doing so it is necessary to know that the G -index of Y is finite. The first aim of this paper is to show that if G is a

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p -torus (resp. torus) group acting without fixed points on a finitistic space X (resp. with finitely many orbit types), then the G -index $i_G(X) < \infty$. This is done in Section 2. Then, as an application, we obtain in Section 5, a generalization of the Central Point Theorem and also of the Tverberg Theorem for any d -dimensional Hausdorff space. The cohomology groups of the classifying spaces of these groups have a neat presentation with a polynomial part in several variables which can be used as a multiplicative set in understanding the scalar product.

2. PRELIMINARIES

Let G be a compact Lie group acting continuously on a topological space X . Suppose B_G denotes the classifying space of G and $E_G \rightarrow B_G$ is the principal G -bundle of G . We have two continuous maps π_1, π_2 defined by the Borel construction

$$\begin{array}{ccccc} X & \leftarrow & X \times E_G & \rightarrow & E_G \\ \downarrow & & \downarrow & & \downarrow \\ X \backslash G & \xleftarrow{\pi_2} & X \times_G E_G & \xrightarrow{\pi_1} & B_G \end{array}$$

Let H^* denote the Alexander-Spanier cohomology and $H_G^*(X) = H^*(X \times_G E_G)$, $H_G^*(pt) = H^*(B_G)$ denote the corresponding equivariant cohomology of the G -space X and the trivial G -space $\{pt\}$ consisting of a point. Then $\Lambda^* = H_G^*(pt)$ is a commutative ring with identity and $H_G^*(X)$ becomes a Λ^* -module via the ring homeomorphism induced by the equivariant map $X \rightarrow \{pt\}$. The Serre spectral sequence of the bundle map $\pi_1 : X_G \rightarrow B_G$ with fibre X defines a sequence of homeomorphism

$$\Lambda^* \rightarrow E_2^{*0} \rightarrow \cdots \rightarrow E_r^{*0} \rightarrow \cdots \rightarrow E_\infty^{*0} \subset H_G^*(X)$$

The kernel of the composite map $\Lambda^* \rightarrow E_\infty^{*0}$ or $\Lambda^* \rightarrow H_G^*(X)$, denoted by $\text{Ind}^G(X)$, is called the “ideal index” of the G -space X . The kernel of the homeomorphism $\Lambda^* \rightarrow E_{r+1}^{*0}$, denoted by ${}_r\text{Ind}^G(X)$, is called the r^{th} filtration of the ideal index $\text{Ind}^G(X) = \bigcap_{r=0}^\infty {}_r\text{Ind}^G(X)$. Now we have (see [13])

Definition 2.1. Let $\alpha (\neq 0) \in \text{Ind}^G(X)$ and $i_X(\alpha)$ denote the smallest r such that $\alpha \in {}_r\text{Ind}^G(X)$. Then $\min\{r \mid i_X(\alpha) = r \text{ and } \alpha (\neq 0) \in \text{Ind}^G(X)\}$ is called the G -index of the G space X and is denoted by $i_G(X)$. Similarly, $\min\{r \mid i_X(\alpha) = r \text{ and } \alpha (\neq 0) \in \text{Ind}^G(X) \text{ and is not a zero-divisor in } \Lambda^*\}$ is yet another G index of X and is denoted by $i'_G(X)$.

Recall that the Yang's homology index, $\text{ind}_G(X)$, defined by using the equivariant homology theory with \mathbb{Z}_2 -coefficients is a well-known numerical \mathbb{Z}_2 -index [15]. Clearly $i_G(X) \leq i'_G(X)$. If $\text{Ind}^G(X) = 0$, we put $i_G(X) = \infty$. Note that if X has a fixed point, then there is a monomorphism from $\Lambda^* \rightarrow H_G^*(X)$ and hence $i_G(X) = \infty$. If $f : X \rightarrow Y$ is a G -map, then $i_G(X) \leq i_G(Y)$. Also, if G acts on the sphere \mathbb{S}^k without fixed point, then $i_G(\mathbb{S}^k) = k + 1$. It is obviously interesting to ask as to what are those G -spaces X on which G acts without fixed points and for which $i_G(X)$ is finite. It must be pointed out that the index $i_G(X)$ of a G -space X is essentially dependent on the action of G on X , e.g., when the circle group $G = \mathbb{S}^1$ acts on an n -sphere \mathbb{S}^n with fixed points, then $i_G(X) = \infty$, but when it acts without fixed points then $i_G(\mathbb{S}^n) = n + 1$.

Here we show that if the group G is a p -torus \mathbb{Z}_p^r or a torus T^r (acting with finitely many orbit types) and X is a finitistic G -space, then with suitable coefficient groups for the cohomology, $i_G(X)$ and $i'_G(X)$ both are finite. The result is already known when X is a compact G -space or X is a paracompact G -space of finite cohomological dimension and G is acting with finitely many orbit types (see [13], Section 3).

Let us recall (see [3], [5]) that a paracompact Hausdorff space X is said to be **finitistic** if each open cover of X has a finite dimensional open refinement. Every compact space is trivially finitistic and every paracompact space of finite covering dimension is also clearly finitistic. On the other hand, there are numerous finitistic spaces which are neither compact nor of finite cohomological dimension, e.g., the product space $X = (\prod_{n=1}^{\infty} \mathbb{S}^n) \times \mathbb{R}^k$. Finitistic spaces are the most general class of spaces suitable for cohomology theory of topological transformation groups ([1],[3]). Hence it is natural to ask whether or not the index $i_G(X)$ is finite when G is acting on a finitistic space X without fixed points.

3. INDEX OF A FINITISTIC G -SPACE IS FINITE

Suppose the group $G = \mathbb{Z}_p^r$ or T^r and the coefficients are in the fields $k = \mathbb{Z}_p$ or \mathbb{Q} respectively. Then we know that (see [7], p.45)

$$\begin{aligned}
H^*(B_G, k) &= k[t_1, t_2, \dots, t_r], \deg t_i = 2(\text{resp. } 1), \\
&\quad \text{when } G = T^r(\text{resp. } \mathbb{Z}_2^r), k = \mathbb{Q}(\text{resp. } \mathbb{Z}_2). \\
H^*(B_G, k) &= k[t_1, t_2, \dots, t_r] \otimes \Lambda[v_1, \dots, v_r], \deg v_i = 1, t_i = \beta(v_i) \\
&\quad \text{when } G = \mathbb{Z}_p^r, \mathbb{Q} = \mathbb{Z}_p \text{ and } p \text{ is odd.}
\end{aligned}$$

Suppose G acts on a finitistic space X without fixed points. For each $x \in X$, the inclusion map $G(x) \rightarrow X$ induces a continuous map $j_x : G(x)_G \rightarrow X_G$ in the Borel constructions. Let $j_x^* : H_G^*(X) \rightarrow H_G^*(G(x))$ denote the induced homomorphism in equivariant cohomology. Also, the constant G -map $G(x) \rightarrow \{pt\}$ induces a homomorphism $H^*(B_G) \rightarrow H^*(G(x)) = H^*(B_G)$ for each $x \in X$. Let $S = R - \{0\}$ be the multiplicative subset where R is the polynomial part of the ring $H^*(B_G, k)$. If G acts on X without fixed points, then G_x is a proper subtorus of G for each $x \in X$, and so from the cohomology algebra mentioned above it follows that for each x , there is an $s \in S$ which goes to zero under the map $H^*(B_G) \rightarrow H^*(B_{G_x})$. Note that $H^*(B_{G_x})$ depends only on the conjugacy class of G_x in G .

If $G = \mathbb{Z}_p^r$, then trivially there are only finitely many conjugacy classes of G_x , but if $G = T^r$, we **assume** that there are only finitely many orbit types in X , and hence in this case also there are only finitely many conjugacy classes of G_x in G . Choosing one $s_i \in S$ for each conjugacy class G_{x_i} , $i = 1, 2, \dots, n$, we find that the product $s = s_1.s_2 \cdots s_n \in S$ has the property that s goes to zero under the map $H^*(B_G) \rightarrow H^*(B_{G_x})$, $\forall x \in X$. If $\pi_1^*(s) \in H_G^*(X)$, we will show that $(\pi_1^*(s))^N = 0$ for some integer N , which will prove that $i_G(X) < \infty$.

Let us call the element $j_x^*(\pi_1^*(s))$ as the **restriction** of the element s on the orbit $G(x)$ and denote it as $s|G(x)$. Since $s|G(x) = 0$ we find by the tautness property of the Alexander-Spanier cohomology that there is an invariant open neighborhood $U(x)$ of x in X such that $s|U(x) = 0$. Thus we have now an open covering $\{U(x)\}$ of the space X . Since X is finitistic, we find from [5] that the orbit space X/G is also finitistic. Hence the open covering $\{q(U(x))\}$ has a finite dimensional open refinement, say $\mathcal{V} = \{V_\alpha \mid \alpha \in \mathcal{A}\}$. Here $q : X \rightarrow X/G$ is the quotient map. Let $\{f_\alpha \mid \alpha \in \mathcal{A}\}$ be a partition of unity subordinate to \mathcal{V} and $\mathcal{W} = \{W_\alpha \mid W_\alpha = f_\alpha^{-1}(0, 1] \subset V_\alpha\}$

Then \mathcal{W} is a finite-dimensional locally finite refinement of \mathcal{V} . Let $\{X_\alpha \mid \alpha \in \mathcal{A}\}$ be a shrinking of $\{W_\alpha\}$ which means $\{X_\alpha \mid \alpha \in \mathcal{A}\}$ is a locally finite, finite-dimensional covering of X/G such that $\overline{X}_\alpha \subset W_\alpha$ for each $\alpha \in \mathcal{A}$. Put

$$Z_\alpha = q^{-1}(X_\alpha) \text{ and } Y_\alpha = q^{-1}(W_\alpha).$$

Then $\{Z_\alpha\}$ and $\{Y_\alpha\}$ both are finite-dimensional locally finite coverings of the space X consisting of invariant open sets so that $\{Z_\alpha\}$ is a shrinking of $\{Y_\alpha\}$

Now for each $\alpha \in \mathcal{A}$, let $\nu_\alpha : X/G \rightarrow [0, 1]$ be a continuous function (X/G is normal) such that $\nu_\alpha(\overline{Z}_\alpha) = 1$ and $\nu_\alpha(X/G - Y_\alpha) = 0$. Let μ_α be the composite map $\nu_\alpha \circ q : X \rightarrow [0, 1]$. Then $\mu_\alpha(\overline{X}_\alpha) = 1$ and $\mu_\alpha(X - W_\alpha) = 0$. Now for any finite nonempty subset F of \mathcal{A} , let

$$U(F) = \{x \in X \mid \min_{\alpha \in F} \mu_\alpha(x) > \max_{\alpha \notin F} \mu_\alpha(x)\}$$

Then $U(F) \subseteq \bigcap_{\alpha \in F} W_\alpha$, and because of finite dimensionality of $\{V_\alpha\}$, there is an integer n such that $U(F) = \emptyset$ whenever $|F| \geq n$. Since $\{V_\alpha\}$ is locally finite, and $\{W_\alpha\}$ is a covering, $\{U(F) \mid F \subset \mathcal{A} \text{ and } |F| < \infty\}$ is an invariant open cover of X . Moreover $|F_1| = |F_2|$ and $F_1 \neq F_2$ implies $U(F_1) \cap U(F_2) = \emptyset$ because otherwise for some $x \in U(F_1) \cap U(F_2)$, we will have

$$\begin{aligned} \min_{\alpha \in F_1} \mu_\alpha(x) &> \max_{\alpha \notin F_1} \mu_\alpha(x) \\ &\geq \min_{\alpha \in F_2} \mu_\alpha(x) \\ &\geq \max_{\alpha \notin F_2} \mu_\alpha(x) \\ &\geq \min_{x \in F_1} \mu_\alpha(x), \end{aligned}$$

a contradiction.

Let $U_i = \bigcup \{U(F) \mid |F| = i\}$. Then $\{U_1, U_2, \dots, U_n\}$ is a covering of X by invariant open sets. Since, for each i , U_i is a disjoint union of $U(F_i)$, we have

$$H_G^*(U_i) = \Pi_{|F|=i} H_G^*(U(F))$$

Also, since $U(F) \subset \bigcap_{\alpha \in F} V_\alpha \subset V_\alpha$ for some α , which is contained in $U(x)$, we find that $s|_F = 0$ for each F , and therefore $s|_{U_i} = 0 \quad \forall i = 1, 2, \dots, n$. Considering the pair (X, U_i) we observe that $\pi_1^*(s) \in H_G^*(X, U_i)$ and hence

the product $\pi_1^*(s^n) = (\pi_1^*(s))^n \in H_G^*(X, X) = 0$. Therefore $(\pi_1^*(s))^n$ is zero in $H_G^*(X)$. We have thus proved the following.

Theorem 3.1. *Suppose $G = \mathbb{Z}_p^r$ (resp T^r) is a p -torus (resp. torus) acts on a finitistic space X without fixed points. If $G = T^r$, we assume further that G acts on X with finitely many orbit types. Then with coefficients in \mathbb{Z}_p (resp. \mathbb{Q}), the index $i_G(X)$ of X is finite.*

The following result follows easily from definitions (see [13]).

Proposition 3.1. *Let X and Y be two G -spaces.*

- (i) *If $\tilde{H}_i(X) = 0$ for $i < N$, then $i_G(X) \geq N + 1$.*
- (ii) *If $H^i(Y) = 0$ for $i > N - 1$ and $i_G(Y) < \infty$, then $i_G(Y) \leq N$.*

We already know that if there is a G -map $f : X \rightarrow Y$, the $i_G(X) \leq i_G(Y)$. Hence we have the following

Corollary 3.1. *Let G be a p -torus acting on spaces X, Y without fixed points. Suppose $\tilde{H}^i(X, \mathbb{Z}_p) = 0$ for $i < N$ and Y is a finitistic space such that $H^i(Y, \mathbb{Z}_p) = 0$ for $i > N - 1$. Then there is no G map from X to Y .*

Corollary 3.2. *Let G be a torus acting on spaces X and Y without fixed points. Assume that $H^i(X, \mathbb{Q}) = 0$ for $i < N$ and Y is finitistic having only finitely many orbit types such that $H^i(Y, \mathbb{Q}) = 0$ for $i > N - 1$. Then there is no G -map from X to Y .*

Remark 3.1. *The condition that the space X is finitistic in the above theorem is needed. For example take any group G and let $E_G \rightarrow B_G$ be the principal G -bundle with E_G as the bundle space. Then G acts on E_G without fixed points (freely) and so the number of orbit types is finite. Since the space $(E_G)_G$ is paracompact and the fibres of $p : (E_G)_G \rightarrow B_G$ are acyclic, the Vietoris-Begle theorem implies that $p^* : H^*(B_G) \rightarrow H^*(E_G)$ is an isomorphism, i.e., $i_G(E_G) = \infty$. The space E_G is clearly nonfinitistic.*

Remark 3.2. *The above theorem is true not only for the groups \mathbb{Z}_p^r and T^r , but also for any compact connected Lie group G provided the coefficients are taken in a field k of characteristic zero, e.g., $k = \mathbb{Q}$. In this case*

$$\ker p^* = \bigcap_{x \in X} \ker j_x^* p^* = \bigcap_{i=1}^m \ker j_{x_i}^* p^*,$$

where the number of orbit types is finite because $\alpha \in \ker j_x^* p^*$ for each $x \in X$ means $p^*(\alpha) \in \ker j_x^*$ for each $x \in X$, and hence there is an integer n such that $(p^*(\alpha))^n = 0$, i.e., $\alpha^n \in \ker p^*$. But $\ker p^*$ is a prime ideal implies $\alpha \in \ker p^*$.

Now if $\ker p^* = 0$, then $\cap_{i=1}^m \ker j_{x_i}^* = 0$. Hence there is an index k such that $\ker j_{x_k} p^* = 0$. This means the map $H^*(B_G) \rightarrow H^*(B_{G_{x_k}})$ is injective map, i.e., $G_{x_k} = G$, which means x_k is a fixed point, a contradiction. Hence $i_G(X) < \infty$.

4. A GENERALIZATION OF THE TOPOLOGICAL CENTRAL POINT THEOREM

The classical central point theorem says that “given any point set X having $m = (d+1)(r-1) + 1$ points in \mathbb{R}^d , there exists a point x in \mathbb{R}^d which is contained in the convex hull of every subset F of X having at least $d(r-1)+1$ points.” The point x is called a central point of X . This theorem has a nontrivial generalization known as the Tverberg Theorem which says that “given any finite set X in \mathbb{R}^d having $m = (d+1)(r-1) + 1$ points, we can partition the set X into r subsets X_1, X_2, \dots, X_r such that

$$\bigcap_{i=1}^r \text{conv}(X_i) \neq \emptyset.$$

Both of these theorems have been generalized in various directions ([3] [11], etc). The following is a recent generalization of the central point theorem [8].

Theorem 4.1. (Topological Central Point Theorem)

Let $m = (d+1)(r-1)$ and Δ^m be the m -simplex. Suppose W is any d -dimensional metric space and $f : \Delta^m \rightarrow W$ is any continuous map then

$$\bigcap_{\dim F = d(r-1)} f(F) \neq \emptyset$$

where F runs over all faces of Δ^m of dimension $d(r-1)$.

The following topological Tverberg theorem is also proved in the same paper [8] for the case when r is a prime power. For general r the theorem is still open. In contrast to this, the topological central point theorem is true for any r .

Theorem 4.2. (Topological Tverberg Theorem)

Let $m = (d+1)r-1$ where r is a prime power and let Δ^m be a m -dimensional simplex. Suppose $f : \Delta^m \rightarrow W$ is any continuous map to a d -dimensional

metric space W . Then there exists r disjoint faces F_1, \dots, F_r of Δ^m such that

$$\bigcap_{i=1}^r f(F_i) \neq \phi.$$

In what follows we will generalize both of the above theorems for continuous maps $f : \Delta^m \rightarrow W$ where W is any Hausdorff space of dimension d , not necessarily a metric space.

5. GENERALIZATION TO d -DIMENSIONAL HAUSDORFF SPACES

Suppose $\dim X$ denotes the covering dimension of a space X [10]. We know that if A is a closed subspace of X , then $\dim A \leq \dim X$. If P is any m -dimensional polyhedron then $\dim P = m$. We also point out that both of the above theorems are proved in [8] with full detail when the space W is a polyhedron of dimension d , and the proof uses the G -index of a G -space X as discussed earlier in this paper for $G = \mathbb{Z}_2$ or $G = (\mathbb{Z}_p)^r$.

Let us first mention that the topological central point theorem is not valid when the space W is an arbitrary d -dimensional space. As an easy example, let us take wedge (one point union with p as a common point) $W = \Delta^d \vee \Delta^m$ where Δ^d is the usual d -simplex and Δ^m is the usual m -simplex with indiscrete topology on $\Delta^m - \{p\}$. Note that Δ^m with indiscrete topology is 0-dimensional but not Hausdorff. Then the central point theorem cannot be true for the continuous map $f : \Delta^m \rightarrow W$ where $f : \Delta^m \rightarrow \Delta^d \vee \Delta^m$ is the obvious inclusion map into the the second part, because the intersection of all $d(r-1)$ dimensional faces of Δ^m is clearly empty.

We have now

Theorem 5.1. *Let $f : \Delta^m \rightarrow W$ be a continuous map where $m = (d+1)(r-1)$ and W be a d -dimensional Hausdorff space. If the topological central point theorem is true for a d -dimensional polyhedron, then the theorem is true for any Hausdorff space W of dimension d .*

Proof. Since the map $f : \Delta^m \rightarrow W$ is continuous and W is Hausdorff, $f(\Delta^m)$ is a compact closed subspace of W . Hence $\dim f(\Delta^m) \leq d$. Moreover, since $f : \Delta^m \rightarrow f(\Delta^m)$ is a perfect map, viz., the inverse image $f^{-1}(p)$ is compact for each p in $p(W)$, the space $f(\Delta^m)$ is also metrizable (see [10] p. 96, Cor

5.8). Thus the space $M = f(\Delta^m)$ is a compact metric space of dimension at most d . Let us denote the map f as the composite

$$\Delta^m \xrightarrow{f'} M \xrightarrow{i} W$$

Since M is a compact metric space of dimension at most d , given any $\epsilon > 0$, we can find a polyhedron P_ϵ of dimension at most d and a map $g_\epsilon : M \rightarrow P_\epsilon$ (see [6], p 322, 6.P.51) such that the $\text{diam}(g_\epsilon^{-1}(p)) \leq \epsilon$ for each $p \in P_\epsilon$.

$$\begin{array}{ccccc} \Delta^m & \xrightarrow{f'} & M & \begin{array}{l} \nearrow i \\ \searrow g_\epsilon \end{array} & \begin{array}{l} W \\ P_\epsilon \end{array} \end{array}$$

Now we know that the theorem is true for the map $g_\epsilon \circ f' : \Delta^m \rightarrow P_\epsilon$ for any given ϵ . This means $\exists p \in P_\epsilon$ such that

$$p \in \bigcap g_\epsilon \circ f'(F),$$

where F runs over all faces of Δ^m of dimension $(d+1)r$. This implies $\cap f'(F) \neq \phi$. To see this suppose F_1, F_2, \dots, F_k are all the $(d+1)r$ dimensional faces of Δ^m . Suppose $f'(F_1), \dots, f'(F_n)$, $n < k$ intersect and there exists an image set, $f'(F_{n+1})$ which is disjoint from $f'(F_1) \cap \dots \cap f'(F_n)$. Let

$$\delta = d(f'(F_{n+1}), \bigcap_{i=1}^n f'(F_i)) > 0.$$

Choose $\epsilon < \frac{\delta}{2}$, a polyhedron P_ϵ and a continuous map $g_\epsilon : M \rightarrow P_\epsilon$ such that $\text{diam}(g_\epsilon^{-1}(p)) < \epsilon$, $\forall p \in P_\epsilon$. Since $\cap_{i \leq k} g_\epsilon \circ f'(F_i) \neq \phi$ we find that $\cap_{i \leq n+1} g_\epsilon \circ f'(F_i) \neq \phi$, and so there is a point $x_1 \in \cap_{i=1}^n f'(F_i)$ and a point $x_2 \in f'(F_{n+1})$ such that $g(x_1) = g(x_2) = p \in P_\epsilon$. But this is a contradiction to the fact that $\text{diam}(g_\epsilon^{-1}(p)) < \epsilon$. Hence $\cap g_\epsilon \circ f'(F) \neq \phi$ and this means $\cap f(F) \neq \phi$. \square

By a similar argument one can prove the Topological Tverberg Theorem also. Since the topological central point theorem and the topological Tverberg theorem both are already known for polyhedra [8], we have the following:

Corollary 5.1. (The Topological Central Point Theorem for Hausdorff spaces): *Let $m = (d+1)(r-1)$ and Δ^m be a m -simplex. Suppose W is a d -dimensional Hausdorff space and $f : \Delta^m \rightarrow W$ is a continuous map. Then if F runs over all the faces of Δ^m of dimension $d(r-1)$, then*

$$\bigcap_{\dim F=d(r-1)} f(F) \neq \emptyset.$$

Corollary 5.2. (The Topological Tverberg Theorem for Hausdorff Spaces): *Let $m = (d+1)r - 1$ where r is a prime power and Δ^m be a m -simplex. Let $f : \Delta^m \rightarrow W$ be a continuous map into a d -dimensional Hausdorff space W . Then there exist r disjoint faces F_1, \dots, F_r of Δ^m such that*

$$\bigcap_{i=1}^r f(F_i) \neq \emptyset.$$

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REFERENCES

- [1] Allday, C. and Puppe, V. *Cohomological methods in transformation groups*, Cambridge Univ. Press, 1993.
- [2] Bárány, I., Shlosman, S. B. and Szűcs, A., *On a topological generalization of a theorem of Tverberg*, J. London Math. Soc. **23** (1981) 158-164.
- [3] Bredon, G. E., *Introduction to compact transformation groups*, Academic Press, New York, 1972.
- [4] Conner, P.E. and Floyd, E.E., *Fixed point free involutions and equivariant maps*, Bull Amer. Math Soc. **66** (1960), 416-441.
- [5] Deo, Satya and Tripathi, H. S., *Compact Lie group actions on finitistic spaces*, Topology **21** (1982) 393-399.
- [6] Engelking, R. and Seikluchi, K., *Topology, A geometric approach*, Overseas Press, 2005.
- [7] Hsiang, Wu Yi, *Cohomology theory of topological transformation groups*, Springer-Verlag, New York, 1975.
- [8] Karasev, R. N., *A topological central point theorem*, Topology and its Applications **159** (2012) 864-868.
- [9] Matoušek, J. *Using the Borsuk-Ulam Theorem*, Springer-Verlag, Universitext, 2003.
- [10] Pears, A. R., *Dimension theory of general spaces*, Cambridge University Press. 1975.

- [11] Sarkaria, K. S., *Tverberg's Theorem via number fields*, Israel J. Math. **79** (1992) 317-320.
- [12] Tverberg, H., *A generalization of Radon's Theorem*, J. London Math. Soc. **41** (1966) 123-128.
- [13] Volovikov, A. Yu., *On the index of G -spaces*, Mat Sbornik **191** (9) (2000) 13-22 (Russian)., Translated in Sbornik Math. **191** (9) (2000) 1255-1277.
- [14] Volovikov, A. Yu., *Coincidence points of functions from \mathbb{Z}_p^k -spaces to CW -complexes*, Russian Math Surveys **57** (1) (2002), 170-172.
- [15] Yang, C.T., *On theorems of Borsuk-Ulam, Kakutani-Yamabe-Yujobo and Dyson II*, Ann. of Math. (2) **(62)** (2), (1955), 271-283.

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